Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 13, 2002*
Accepted: *January 19, 2003*

Derivative corrections in 10-dimensional super-Maxwell theory

Andres Collinucci, Mees de Roo and Martijn G.C. Eenink

Institute for Theoretical Physics
Nijenborgh 4, 9747 AG Groningen, The Netherlands
E-mail: a.collinucci@phys.rug.nl, m.de.roo@phys.rug.nl, m.g.c.eenink@phys.rug.nl

Abstract: We construct the supersymmetric effective action at order α'^4 of the abelian open superstring. It includes the α'^4 terms in the abelian Born-Infeld action, and in particular the leading derivative correction of the form $\partial^4 F^4$. Besides linear supersymmetry this sector of the open string effective action also has a nonlinear supersymmetry. The terms $\partial^4 F^4$ and their fermionic partners have an arbitrary coefficient, and we discuss the possible fate of such coefficients when higher orders in α' are included.

Keywords: Superstrings and Heterotic Strings, D-branes, Supersymmetric Effective Theories.

JHEP01(2003)039

Contents

1. Introduction **1**

2. Constructing α' corrections **3**

3. Results **4**
3.1 Orders α'^1, α'^2 and α'^3 4
3.2 Order α'^4 5
3.3 Nonlinearly realised SUSY 6
3.4 Closure of the algebra 6

4. Discussion and conclusions **7**

A. Transformation rules **10**
A.1 Linear supersymmetry 10
A.2 Nonlinear supersymmetry 12

1. Introduction

The tree-level effective action of the open string, with or without Chan-Paton factors, has drawn a lot attention recently [1]. Without Chan-Paton factors it corresponds, for slowly varying fields, to the Born-Infeld action [2]. Its supersymmetric completion can be obtained quite elegantly using κ-symmetry [3]–[6]. With Chan-Paton factors the action is known only for some low orders of α' [7]–[13]. The complications in this case are partly due to the fact that for a nonabelian gauge theory $[\mathcal{D},\mathcal{D}]F = [F,F]$, so that the approximation in which derivatives of the fields are ignored can no longer be made. That such derivative terms are also present in the abelian case is clear from string scattering amplitudes, e.g. the four-point function, as mentioned in the early papers on the open string effective action [2, 19]. Nevertheless not much is known about this extension of the Born-Infeld action at the present time.

In this paper we will investigate these higher derivative terms in the context of supersymmetry. The motivation for this is, besides the intrinsic interest in the string effective action, that these terms also have a relation with the nonabelian extension of the Born-Infeld action. One way to approach this problem is by using κ-symmetry, in which linear and nonlinear supersymmetry arise from the gauge fixing of a local fermionic symmetry. Recent efforts in this direction have not been successful [14, 15], but alternatives are under investigation [16]–[18]. It should be realized that the higher-derivative contributions of the *abelian* effective action do not fit in the present κ-symmetric formulation [3]–[6]. Thus a supersymmetric, and eventually a κ-symmetric, formulation of these terms could be helpful in solving the nonabelian problem.

JHEP01(2003)039

In this paper we take a first step in this direction, which is to obtain all terms in the abelian effective action through order α'^4 by imposing supersymmetry. The result is that there are two different independent supersymmetric invariants. The first invariant consists of terms at order α'^2 and α'^4, respectively of the form F^4 and F^6 and their fermionic partners, and is the contribution to the Born-Infeld invariant through this order. The second invariant involves only terms at order α'^4: $\partial^4 F^4$ and their fermionic partners. The terms with two derivatives, $\partial^2 F^5$ and their fermionic counterparts, turn out to be inconsistent with supersymmetry. Furthermore, all conceivable terms at α'^4 with a higher number of derivatives are removable by field redefinitions and therefore trivial.

In discussing the higher derivative contributions to the open string effective action it is useful to introduce some notation. We write such terms as

$$\mathcal{L}_{(m,n)} = \frac{\alpha'^m}{g^2}\left(\partial^n F^p + \partial^{n+1} F^{p-2}\bar{\chi}\gamma\chi\right), \tag{1.1}$$

where g is the gauge coupling constant of dimension $-(d-4)/2$. Henceforth we will set $g = 1$. The powers in (1.1) are related by $2p - 2m + n - 4 = 0$. We will denote the terms at order α'^m and with n derivatives by (m, n). In this paper we do not consider fermion-dependent contributions beyond the bilinear fermion terms in (1.1).

The Noether procedure we employ to find the supersymmetric deformations of the super-Maxwell action is an iterative procedure. At a given order in α' it yields a number of apparently independent superinvariants, all determined up to a multiplicative constant. For example, the $(4, 4)$ terms we will discuss have an arbitrary coefficient $a_{(4,4)}$ that is not fixed by supersymmetry. However, some of these coefficients might be determined by pursuing the Noether procedure for higher values of m and n. One can also use input from string theory. We will limit ourselves to the contributions to the effective action that follow from the open string tree-level S-matrix. The tree-level correlation functions that one derives from the effective action should reproduce these string amplitudes, which allows one to fix the previously undetermined coefficients. In particular, the $(m, 2m-4)$ terms should reproduce the α'-expanded 4-point function. For m odd, this expansion contains a coefficient $\pi^2\zeta(m-2)$. It is hard to see how the Noether procedure could determine such coefficients in the absence of algebraic relations between the values of the Riemann zeta-function for odd integer arguments. Therefore these terms should all correspond to separate independent superinvariants, as we argued in [11]. We will come back to these points in the discussion.

Most of the information on derivative corrections concerns bosonic terms only. In [21] it was shown that terms $(m, 2)$ vanish for all m. Our calculation of $(4, 2)$ confirms this and extends it to the corresponding fermionic terms. In [21] the bosonic part of $(4, 4)$ was constructed explicitly. More recently, Wyllard [22] obtained the $(m, 4)$ terms using the boundary state formalism. Further work has been done in [23, 24] with the Seiberg-Witten map and noncommmutativity. Information about the fermionic contributions can in principle be obtained from calculating superstring scattering amplitudes involving external fermions. The required formalism can be found in [25], and applications using the four-point function to orders α'^m, $m \leq 4$ can be found in [15, 26]. The recent determination of the string five-point function and its relation with the nonabelian Born-Infeld action [12] concerns bosonic terms at order α'^3 only.

This paper is organized as follows. In section 2 we will briefly discuss our method, the main results on linear and nonlinear supersymmetry are given in section 3. Finally, in section 4, we discuss the general structure of the web of supersymmetric derivative corrections.

2. Constructing α' corrections

In this section we review our method of imposing supersymmetry order by order in α'. A more detailed exposition of this iterative (or Noether) procedure can be found in [11]. Starting point is the $d = 10$, $N = 1$ supersymmetric Maxwell lagrangian[1]

$$\mathcal{L}_0 = -\frac{1}{4} F_{ab} F_{ab} + \frac{1}{2} \bar{\chi} \partial\!\!\!/ \chi. \tag{2.1}$$

The equations of motion are simply

$$\partial_a F_{ab} = \partial\!\!\!/ \chi = 0\,, \tag{2.2}$$

supersymmetry is realised linearly on the fields:

$$\begin{aligned} \delta_0 A_a &= \bar{\epsilon} \gamma_a \chi\,, \\ \delta_0 \chi &= \frac{1}{2} F_{ab} \gamma_{ab} \epsilon\,. \end{aligned} \tag{2.3}$$

Closure of the supersymmetry algebra requires the fields to be on-shell and involves a field dependent gauge transformation of the gauge field:

$$\begin{aligned} [\delta_{0\,\epsilon_1}, \delta_{0\,\epsilon_2}] A_a &= 2\bar{\epsilon}_1 \partial\!\!\!/ \epsilon_2 A_a - \partial_a (2\bar{\epsilon}_1 A\!\!\!/ \epsilon_2)\,, \\ [\delta_{0\,\epsilon_1}, \delta_{0\,\epsilon_2}] \chi &= 2\bar{\epsilon}_1 \partial\!\!\!/ \epsilon_2 \chi - \left(\frac{7}{8} \bar{\epsilon}_1 \gamma_a \epsilon_2 \gamma_a - \frac{1}{5!16} \bar{\epsilon}_1 \gamma_{abcde} \epsilon_2 \gamma_{abcde} \right) \partial\!\!\!/ \chi\,. \end{aligned} \tag{2.4}$$

This lowest order action also has a nonlinear supersymmetry:

$$\begin{aligned} \delta_0 A_a &= 0\,, \\ \delta_0 \chi &= \eta\,. \end{aligned} \tag{2.5}$$

The iterative procedure consists of two steps. Let the $\mathcal{L}_k$ for $k < m$ be known. The first step in obtaining the term $\mathcal{L}_m$ is to write down all possible terms of order α'^m, i.e., terms that have the correct dimension and are Lorentz and gauge invariant. We limit ourselves to terms that are at most of quadratic order in the fermions. lagrangians are defined up to

[1]We work in Minkowski spacetime and write spacetime indices as lower indices. Our conventions for the γ-matrices follow [27]. χ is a Majorana-Weyl spinor and inert under gauge transformations. $\mathcal{L}_m$ is the contribution of order α'^m to the effective action. Similarly, δ_m indicate supersymmetry transformations of order α'^m. If we want to indicate the part of $\mathcal{L}_m$ with n derivatives we write $\mathcal{L}_{(m,n)}$, similarly for $\delta_{(m,n)}$.

total derivatives and field redefinitions. The possibility for the latter arises when a term is proportional to the lowest order equation of motion (2.2) for one of the fields. If such a term is present in $\mathcal{L}_m$ it can be removed by a field redefinition of order m. The price one pays is that the contributions $\mathcal{L}_n$ with $n > m$ are modified. We deal with this ambiguity, at each order in α', by not allowing in the lagrangian any terms that are proportional to the order α'^0 field equations, or terms that can be rewritten as such by means of a partial integration. Furthermore, we determine how the remaining terms are related by partial integrations and keep only an independent subset. This leaves us with a minimal Ansatz for $\mathcal{L}_0$ in which each term has an arbitrary coefficient that will be determined in the second step.

The second step is to vary the fields in this Ansatz with the supersymmetry transformation rules δ_0. In addition we need to vary the lower order terms in the lagrangian, say $\mathcal{L}_k$, $k < m$, with the appropriate transformation rules δ_{m-k}; both were already obtained in a previous stage of the iterative procedure. Having done this, we are left with two types of variations. On the one hand there are terms which are proportional to the lowest order field equation or that can be rewritten as such using a partial integration. On the other hand there are variations that cannot be rewritten in this way. The first set can be eliminated by new transformation rules δ_m of $\mathcal{L}_0$, the second set must be set to zero by solving the resulting equations for the unknown coefficients in the Ansatz.

In calculating the new transformation rules at order α'^m one will find that some variations may be quadratic in the lowest order equations of motion. In that case there is an ambiguity in the choice of the new transformations δ_m. Regardless of this choice, such variations always give rise to transformation rules that contain a lowest order equation of motion. Therefore these terms do not play a role in checking the closure of the supersymmetry algebra at order α'^m. If such transformations are applied to some $\mathcal{L}_k$ when constructing an invariant at order $m + k$, they give variations that can automatically be supersymmetrized. Their contribution to the order $m + k$ transformation rules need not contain a lower order equation of motion and therefore these terms are important when pursuing the Noether procedure to higher orders. Note however that this last issue does not yet play a role at order α'^4 and should not bother us in this paper.

The procedure is applied for both linear and nonlinear supersymmetry.

3. Results

In the end we are left with all possible deformations of the lagrangian and the supersymmetry tranformation rules at a certain order in α', up to field redefinitions. In this section we will give the action and study the algebra of the linear and nonlinear supersymmetry transformations. The transformation rules themselves are given in appendix A.

3.1 Orders α'^1, α'^2 and α'^3

It is well known that there are no nontrivial supersymmetric deformations of (2.1) at order α'^1, i.e. all terms allowed by supersymmetry can be removed by field redefinitions. In [28] the terms at order α'^2 were obtained by using the Noether procedure. For completeness, and since we need these results at order α'^4, we review them here.

JHEP01(2003)039

Following the steps outlined in the previous section, one first writes down an Ansatz for the lagrangian; these are all terms of the form F^4, $\partial^2 F^3$, $\partial^4 F^2$, $\partial^6 F$ and fermionic partners. All of these terms turn out to be removable by field redefinitions, except for F^4 and its fermionic counterpart $\partial F^2 \bar{\chi}\gamma\chi$: $\mathcal{L}_2 = \mathcal{L}_{(2,0)}$. Imposing supersymmetry fixes the coefficients in this Ansatz up to one overall multiplicative constant. The result is:

$$\mathcal{L}_2 = \frac{a_{(2,0)}\alpha'^2}{32}\{ -F_{ab}F_{ab}F_{cd}F_{cd} + 4\,F_{ac}F_{bd}F_{ab}F_{cd} - \\ - 8\,F_{ab}F_{ac}\,\bar{\chi}\gamma_b\partial_c\chi - 2\,F_{ab}\partial_a F_{cd}\,\bar{\chi}\gamma_{bcd}\chi\}\,. \tag{3.1}$$

Of course, these terms are the same as the ones obtained from the Born-Infeld invariant[2]. The supersymmetry transformations also receive α'^2 contributions; they are given in the appendix A. From the point of view of supersymmetry the coefficient $a_{(2,0)}$ is arbitrary. At tree-level the string four-point function sets $a_{(2,0)} \sim \pi^2$.

At order α'^3 there are no supersymmetric contributions. This might be inferred by taking the abelian limit of the results of [13, 11]. However, since it is not obvious that every supersymmetric abelian action allows a nonabelian supersymmetric extension, it is important to check this directly in the abelian context. This has been done in [20] by superspace methods, and we have verified this result by an independent calculation using the method of section 2.

3.2 Order α'^4

We now turn to the main topic of this paper: the α'^4 contributions. There are three nontrivial sectors in the Ansatz: $(4,0)$, $(4,2)$, $(4,4)$. The structures with more derivatives are removable by field redefinitions. Furthermore, it turns out that this is also the case for the bosonic terms $(4,2)$, i.e., the terms $\partial^2 F^5$, but not for their fermionic partners, which are of the form $\partial^3 F^3 \bar{\chi}\gamma\chi$. In applying the method of section 2 we need the variations $\delta_0\mathcal{L}_{(4,0)}$ as well as $\delta_2\mathcal{L}_{(2,0)}$. In the cases $(4,2)$ and $(4,4)$ only the variation δ_0 is needed.

The results of the Noether procedure are the following: in the sector $\mathcal{L}_{(4,0)}$ with F^6 and $\partial F^4\bar{\chi}\gamma\chi$ the only terms allowed by supersymmetry are those needed for the 'continuation' of the invariant of order α'^2, i.e. the Born-Infeld invariant. Thus there appears no *new* invariant, independent of the lower orders. Furthermore, the fermionic terms in $\mathcal{L}_{(4,2)}$ of the form $\partial^3 F^3\bar{\chi}\gamma\chi$ are not supersymmetrizable. Finally, in the section $\mathcal{L}_{(4,4)}$ there does appear a new invariant. We have verified that the bosonic terms of this invariant are the same as those of [22, 13] up to field redefinitions. We however disagree with [26], where both bosonic and fermionic terms are determined by comparison with the string four-point function.

The action at order α'^4, $\mathcal{L}_4 = \mathcal{L}_{(4,0)} + \mathcal{L}_{(4,4)}$, reads:

$$\mathcal{L}_{(4,0)} = \frac{(a_{(2,0)})^2\alpha'^4}{384}\{ -32\,F_{ab}F_{bc}F_{cd}F_{de}F_{ef}F_{af} - 12\,F_{ab}F_{bc}F_{cd}F_{ad}F_{ef}F_{ef} - \\ - F_{ab}F_{ab}F_{cd}F_{cd}F_{ef}F_{ef} - 12\,\partial_a F_{bc}F_{de}F_{af}F_{be}\,\bar{\chi}\gamma_{cdf}\chi +$$

[2]Up to a field redefinition.

JHEP01(2003)039

$$
\begin{aligned}
&+ 72\, F_{ab}F_{cd}F_{be}F_{de}\, \bar{\chi}\gamma_a\partial_c\chi + 18\, \partial_a F_{bc}F_{de}F_{ef}F_{af}\, \bar{\chi}\gamma_{bcd}\chi + \\
&+ 12\, \partial_a F_{bc}F_{de}F_{bf}F_{ae}\, \bar{\chi}\gamma_{cdf}\chi\}\,, \qquad (3.2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{(4,4)} = a_{(4,4)}\alpha'^4\{ &- 8\, F_{ab}F_{bc}\partial_d\partial_e F_{af}\partial_d\partial_e F_{cf} - 8\, F_{ab}\partial_c F_{ad}\partial_e F_{bf}\partial_c\partial_e F_{df} + \\
&+ 32\, F_{ab}\partial_c F_{ad}\partial_e F_{bf}\partial_d\partial_e F_{cf} + 16\, F_{ab}\partial_c F_{de}\partial_a F_{ef}\partial_d\partial_f F_{bc} + \\
&+ 4\, \partial_a\partial_b F_{cd}\partial_a\partial_b F_{ce}\, \bar{\chi}\gamma_d\partial_e\chi - 4\, \partial_a F_{bc}\partial_a\partial_d F_{ef}\, \bar{\chi}\gamma_{bef}\partial_c\partial_d\chi + \\
&+ 4\, F_{ab}\partial_c\partial_d F_{ef}\, \bar{\chi}\gamma_{abe}\partial_c\partial_d\partial_f\chi + 8\, F_{ab}\partial_c\partial_d F_{ae}\, \bar{\chi}\gamma_b\partial_c\partial_d\partial_e\chi + \\
&+ 2\, \partial_a F_{bc}\partial_a\partial_d\partial_e F_{bc}\, \bar{\chi}\gamma_d\partial_e\chi\} \qquad (3.3)
\end{aligned}
$$

Note that the overall coefficient of $\mathcal{L}_{(4,0)}$ is uniquely fixed by supersymmetry in terms of $a_{(2,0)}$, the coefficient $a_{(4,4)}$ is unrelated. String theory tells us that at tree-level $a_{(4,4)} \sim \pi^4$.

3.3 Nonlinearly realised SUSY

Nonlinear supersymmetry arises from the breaking of $N = 2$ supersymmetry to $N = 1$: the superstring effective action corresponds to the worldvolume theory of a D9-brane, and D-branes break half of the $N = 2$ supersymmetry. In the κ-symmetric formulation of the Born-Infeld action it arises from the gauge-fixing of the local κ-symmetry. The presence of this nonlinear symmetry can be taken as an indication that a κ-symmetric formulation is possible, but there is certainly no proof of such a relation.

The nonlinear supersymmetry is quite restrictive in the sectors without extra derivatives: $(2,0)$ and $(4,0)$. For instance, in $(4,0)$ one finds the result $\mathcal{L}_{(4,0)}$ (3.2), plus one additional term which is invariant under the nonlinear symmetry (up to variations which vanish on-shell) all by itself. It is to be noted that in the sector $(4,2)$ it is possible to impose nonlinear supersymmetry. Similarly, in $(4,4)$ nonlinear supersymmetry is not restrictive at all, and many combinations of terms are invariant under (2.5).

3.4 Closure of the algebra

The supersymmetry algebra can only be evaluated on the vector field, due to the absence of higher fermion terms in the action and transformation rules. We find, for all contributions of order α'^m, $m \leq 4$:

$$
\begin{aligned}
[\delta_{\epsilon_1}, \delta_{\epsilon_2}]A_a = {}& 2\bar{\epsilon}_1 \not{\partial} \epsilon_2 A_a - \partial_a(2\bar{\epsilon}_1 \not{A} \epsilon_2) + \qquad (3.4) \\
&+ a_{(4,4)}\, \partial_a\{ + 32\, \partial_c\partial_d F_{eb}\partial_c F_{df}F_{ef}\, \bar{\epsilon}_1\gamma_b\epsilon_2 + 16\, \partial_c\partial_d F_{eb}\partial_c F_{ef}F_{df}\, \bar{\epsilon}_1\gamma_b\epsilon_2 + \\
&\qquad + 8\, \partial_c\partial_d F_{ef}\partial_c F_{ef}F_{db}\, \bar{\epsilon}_1\gamma_b\epsilon_2 + 16\, \partial_c F_{db}\partial_e F_{fc}\partial_e F_{fd}\, \bar{\epsilon}_1\gamma_b\epsilon_2 + \\
&\qquad - 16\, \partial_c F_{db}\partial_e F_{fc}\partial_f F_{ed}\, \bar{\epsilon}_1\gamma_b\epsilon_2 + 4\, \partial_g\partial_h F_{bc}\partial_g F_{de}F_{hf}\, \bar{\epsilon}_1\gamma_{bcdef}\epsilon_2 + \\
&\qquad + 4\, \partial_g\partial_h F_{bc}\partial_g F_{hd}F_{ef}\, \bar{\epsilon}_1\gamma_{bcdef}\epsilon_2 - 4\, \partial_g F_{bc}\partial_h F_{de}\partial_g F_{hf}\, \bar{\epsilon}_1\gamma_{bcdef}\epsilon_2\}\,.
\end{aligned}
$$

Note that the terms without extra derivatives do not modify the gauge transformation in the algebra. This is simply due to the absence of a derivative in the corresponding transformation rules. Nevertheless, the required cancellations for closure, and the fact that all remaining terms combine into a gauge transformation, is an important check on our result.

The algebra of the nonlinear transformations reads [7]:

$$[\delta_{\eta_1},\delta_{\eta_2}]A_a = \frac{a_{(2,0)}\alpha'^2}{2}\left(\bar\eta_1 \partial\!\!\!/ \eta_2 A_a - \partial_a(\bar\eta_1 A\!\!\!/ \eta_2)\right), \tag{3.5}$$

which does not have modifications at order α'^4. Note that this is just the usual supersymmetry algebra, occurring at a higher order in α'. This *proves* that the nonlinear symmetry is indeed a supersymmetry.

The mixed algebra takes on the form:

$$[\delta_\epsilon,\delta_\eta]A_a = \frac{a_{(4,4)}\alpha'^2}{80}\,\partial_a(\partial_b\partial_c F_{de}(5\partial_c F_{de}\bar\epsilon\gamma_b\eta + 2\partial_b F_{cf}\bar\epsilon\gamma_{def}\eta))\,. \tag{3.6}$$

4. Discussion and conclusions

Our present knowledge of the open string effective action is represented in figure 1. Black dots indicate the sectors (m,n) for which both bosonic and fermionic terms have been established. The dots $(m,0)$ for all m form the Born-Infeld action, for which the result is known to all orders in the fermions [3]–[6]; for the single dot $(4,4)$, presented in this paper, only the terms bilinear in the fermions are known. White dots without diagonal lines are known to be empty. These include all points $(m,2(m-1))$, that would correspond to the three-point function[3]. We have also put a white dot for all points $(m,2)$, although strictly speaking the absence of these contributions has only been established for the bosonic terms.

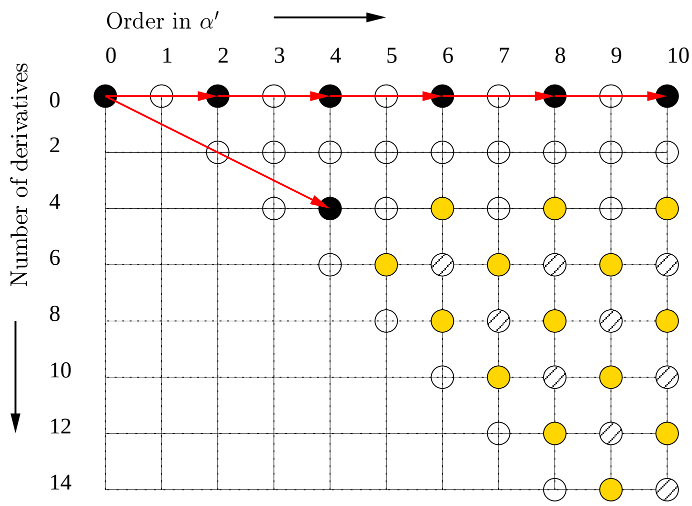

Figure 1: Structure of the abelian open superstring tree level effective action. Black dots indicate nonempty sectors of which the explicit form is known. Empty white dots correspond to sectors that are known to be empty up to field redefinitions, already taking into account conjecture 1. Yellow dots indicate sectors that are known to be nonempty, but have yet to be constructed explicitly. Slashed white dots indicate sectors that should be empty by conjecture 2. The red arrows indicate the known supersymmetry transformation rules.

[3] In addition, it turns out that all terms that one could possibly write down in this sector can be removed by field redefinitions.

As far as we know there is, in this order by order superinvariant, no way to exclude a priori the presence of a fermionic contribution in these sectors. Nevertheless, we make here the first conjecture:

Conjecture 1. If the bosonic part of the sector (m, n) vanishes, then also the fermionic contributions of that sector

The white dots also include all points $(m, 4)$ for m odd. In [22] the bosonic part of $(m, 4)$ was obtained for all m; it vanishes for m odd. In favor of this conjecture is our result for $(4, 2)$. The conjecture implies that the terms $(m, 0)$, m odd, all vanish. This is obvious for the bosonic terms, but it has to be checked for the fermionic terms. This could be done for instance by starting with the explicit form of the Born-Infeld action given by [6] and by doing the field redefinitions needed to eliminate all fermionic contributions at odd orders in α'. For the higher derivative terms a useful check would be $(5, 4)$.

There are many sectors which are known to be present but for which the complete contribution to the effective action is presently unknown. These yellow dots include all points $(m, 2m-4)$ corresponding to the four-point function, all points $(m, 4)$, m even [22], and we would add to these all points $(m, 2m-4k+4)$ (for $m > 2k-1$ to avoid $n = 2$) which correspond to higher derivative contributions to the open string $2k$-point function.

The remaining points are white with diagonal lines, and correspond to contributions to the string $2k+1$-point functions for $k > 1$. Everything we know so far about the effective action would be consistent with the vanishing of these contributions, which would imply the vanishing of the open string $2k+1$-point function without Chan-Paton factors. Note that present knowledge confirms this conjecture for terms with 0, 2 and 4 extra derivatives. Thus we make a second conjecture[4]:

Conjecture 2. The tree level open string *odd*-point function without Chan-Paton factors vanishes.

Since this is a conjecture about string theory, it cannot be checked by using supersymmetry alone. For the effective action it implies that there are no terms with an odd number of fields (either bosons or fermions).

With these conjectures in mind we analyze the supersymmetry transformations that connect the dots in figure 1. We have drawn arrows to indicate the known supersymmetry transformations. As a first example we consider the terms $(m, 0)$, m even, i.e., the Born-Infeld invariant. These terms are invariant under the transformations $\delta_0, \delta_{(2,0)}, \delta_{(4,0)}, \ldots$, depending only on the single parameter $a_{(2,0)}$. Note that we indicate these transformations by a repeated addition of the same arrow, and *not* by drawing new arrows from $(0, 0)$ to $(m, 0)$ for each m. In this way we denote that all these terms contribute to the same invariant. Similarly, the point $(4, 4)$ is the leading term in a new sequence of supersymmetry transformations that continues to the points $(4k, 4k)$, involving the parameter $a_{(4,4)}$. It is clear that all points on the diagonal $(m, 2m-4)$ will lead to at least one new sequence of arrows or supersymmetry transformations, involving parameters $a_{(m,2m-4)}$. The question is now, whether these new 'independent' invariants will remain independent when the

[4]It is a pleasure to thank A. Tseytlin for an interesting correspondence on this point.

Noether procedure is pursued to higher orders. Consider for example the point $(8, 8)$. This point can be reached from $(0, 0)$ by applying the arrow $(4, 4)$ twice, but also by applying the arrow $(2, 0)$ and then $(6, 8)$ (or vice versa). These contributions need to be cancelled by the δ_0 variation of $\mathcal{L}_{(8,8)}$. In principle, there are now two possibilities: they can either be cancelled separately, or not. In the latter case we need both contributions at the same time, and then there must be relations between the coefficients $a_{(4,4)}a_{(4,4)}$ and $a_{(2,0)}a_{(6,8)}$. So it is indeed possible that a priori independent invariants are related to each other at higher orders in the iteration. Note however that at least $a_{(2,0)}$ and $a_{(4,4)}$ will remain independent to all orders. The reason is that $a_{(2,0)}$ and $a_{(4,4)}$ can be changed independently by rescaling α' and the extra derivatives, respectively.

Now we invoke our knowledge of the string tree-level 4-point function [11]. In the tree-level effective action the coefficients $a_{(m,2m-4)}$ for m even contain factors π^m, and therefore the coefficients $a_{(2,0)}$, $a_{(4,4)}$ and $a_{(6,8)}$ are, in string theory, all proportional to powers of π. The relations between these coefficients alluded to in the previous paragraph are therefore possible in the string theory context. Independence of these coefficients would imply that there are more supersymmetric invariants than required by string theory.

At odd m, and also at even m for m large enough, the situation is different. At odd m the string four-point function has a factor $\pi^2\zeta(m-2)$. This implies that the coefficients $a_{(m,2m-4)}$, for m odd, will remain independent to all orders in the Noether procedure, since there are no (known) relations between the values of the Riemann ζ-function for odd values of its argument. At certain even values of m one finds, besides powers of π, also terms with ζ-functions. An interesting example is the point $(10, 12)$. We can reach it with supersymmetry transformations through $(6, 8)$ by applying $\delta_{(4,4)}$, from $(8, 12)$ by applying $\delta_{(2,0)}$, and from $(5, 6)$ by applying $\delta_{(5,6)}$. Now it should be noted that at $(8, 12)$ the string four-point function has two separate kinematic structures: one proportional to π^{10}, and one proportional to $\pi^2\zeta(3)^2$. The three different ways of arriving at $(10, 12)$ therefore lead to two kinds of terms: those with π^{12} and those with $\pi^4\zeta(3)^2$. These must belong to two separate supersymmetric invariants. Again, the minimum requirement is that these are part of the invariants containing $(4, 4)$ and $(5, 6)$, anything else leads to an accumulation of more and more invariants not required by string theory.

The minimum assumption is therefore that supersymmetry requires independent coefficients at $(4, 4)$, and at $(m, 2m-4)$ with m odd, the points where $\pi^2\zeta(m-2)$ appears. This leads to the conjecture:

Conjecture 3. The sectors $\mathcal{L}_{(4,4)}$ and $\mathcal{L}_{(m,2m-4)}$, m odd, contain the leading contributions to separate superinvariants. There are no other invariants starting at $\mathcal{L}_{(m,n)}$ for any m, n.

The independent coefficients in the maximal extension of supersymmetric Maxwell theory in $d = 10$ are, according to these conjectures, $a_{(2,0)}$, $a_{(4,4)}$ and $a_{(m,2m-4)}$ for m odd. The tree-level open string effective action corresponds to a particular choice of these coefficients. The independence of $a_{(2,0)}$ and $a_{(4,4)}$ implies that the Born-Infeld action for slowly varying fields is a separate invariant.

JHEP01(2003)039

The issues that we raised above clearly need to be addressed. It is probably not possible to continue the Noether procedure we used much further, due to the rapidly increasing number of possible terms in the lagrangian at higher orders in α'. One should therefore look for other methods of tackling these issues. In particular, it would be interesting to see whether more information on the structure of the superinvariants can be obtained from string theory considerations. Another possibility would be to set up the Noether procedure in $d = 10$ $\mathcal{N} = 1$ on-shell superspace. A clear advantage of this setting is that field redefinition ambiguities do not arise, since all fields are constrained to satisfy their lowest order equations of motion. Finally, the persistence of the non-linear supersymmetry in the higher-derivative terms is a strong indication that a κ-symmetric formulation of the all-order effective action exists. Given the success of κ-symmetry in clarifying the structure of the supersymmetric Born-Infeld action, it is conceivable, if not likely, that it will yield similar striking results when applied to this problem.

Acknowledgments

We thank Arkady Tseytlin for an interesting correspondence. The work of Martijn Eenink is part of the research programme of the "Stichting voor Fundamenteel Onderzoek van de Materie" (FOM). This work is supported in part by the European Commission RTN programme HPRN-CT-2000-00131, in which we are associated to the University of Utrecht.

A. Transformation rules

In the appendix we will give the complete set of transformation rules for all orders α'^m, $m \leq 4$. As discussed in section 2, we give only those transformations that do not vanish on-shell. These are needed to establish the existence of an invariant, but do not change the supersymmetry algebra.

A.1 Linear supersymmetry

The variation of the gauge field reads:

$$
\begin{aligned}
\delta A_a =\ & \bar\epsilon\gamma_a\chi + \frac{a_{(2,0)}\alpha'^2}{32}\{ - 6\, F_{cd}F_{cd}\,\bar\epsilon\gamma_a\chi - 16\, F_{ac}F_{cd}\,\bar\epsilon\gamma_d\chi - \\
& \qquad - 4\, F_{ac}F_{de}\,\bar\epsilon\gamma_{cde}\chi + F_{cd}F_{ef}\,\bar\epsilon\gamma_{acdef}\chi\} + \\
& + \frac{a_{(2,0)}^2\alpha'^4}{1024}\Big\{ +96\, F_{ab}F_{cd}F_{cd}F_{be}\,\bar\epsilon\gamma_e\chi - 128\, F_{ab}F_{bc}F_{cd}F_{de}\,\bar\epsilon\gamma_e\chi + \\
& \quad + 104\, F_{bc}F_{bd}F_{ce}F_{de}\,\bar\epsilon\gamma_a\chi + 18\, F_{bc}F_{bc}F_{de}F_{de}\,\bar\epsilon\gamma_a\chi + \\
& \quad + 32\, F_{ab}F_{cd}F_{ce}F_{df}\,\bar\epsilon\gamma_{bef}\chi + 24\, F_{ab}F_{cd}F_{cd}F_{ef}\,\bar\epsilon\gamma_{bef}\chi + \\
& \quad + 32\, F_{ab}F_{bc}F_{de}F_{cf}\,\bar\epsilon\gamma_{def}\chi - 16\, F_{ab}F_{cd}F_{be}F_{fg}\,\bar\epsilon\gamma_{cdefg}\chi - \\
& \quad - 6\, F_{bc}F_{bc}F_{de}F_{fg}\,\bar\epsilon\gamma_{adefg}\chi - 16\, F_{bc}F_{bd}F_{ce}F_{fg}\,\bar\epsilon\gamma_{adefg}\chi - \\
& \quad - \frac{4}{3}F_{ab}F_{cd}F_{ef}F_{gh}\,\bar\epsilon\gamma_{bcdefgh}\chi + \frac{1}{6}F_{bc}F_{de}F_{fg}F_{hi}\,\bar\epsilon\gamma_{abcdefghi}\chi\Big\} + \\
& + a_{(4,4)}\alpha'^4\{ + 20\,\partial_b F_{cd}\partial_a\partial_b\partial_e F_{cd}\,\bar\epsilon\gamma_e\chi + 28\,\partial_b\partial_c F_{ad}\partial_b\partial_d F_{ce}\,\bar\epsilon\gamma_e\chi -
\end{aligned}
$$

JHEP01(2003)039

$$
\begin{aligned}
&- 28\,\partial_b\partial_c F_{ad}\partial_b\partial_c F_{de}\,\bar\epsilon\gamma_e\chi + 8\,\partial_b F_{cd}\partial_a\partial_b\partial_c F_{ef}\,\bar\epsilon\gamma_{def}\chi + \\
&+ 2\,\partial_a\partial_b F_{cd}\partial_b\partial_c F_{ef}\,\bar\epsilon\gamma_{def}\chi + 4\,\partial_b\partial_c F_{de}\partial_b\partial_c F_{fg}\,\bar\epsilon\gamma_{adefg}\chi + \\
&+ 4\,F_{bc}\partial_a\partial_d\partial_e F_{bc}\,\bar\epsilon\gamma_d\partial_e\chi + 12\,\partial_b F_{cd}\partial_b\partial_e F_{cd}\,\bar\epsilon\gamma_a\partial_e\chi + \\
&+ 4\,\partial_b F_{cd}\partial_b\partial_e F_{cd}\,\bar\epsilon\gamma_e\partial_a\chi - 84\,\partial_b F_{cd}\partial_a\partial_c F_{be}\,\bar\epsilon\gamma_d\partial_e\chi + \\
&+ 72\,\partial_b F_{cd}\partial_a\partial_b F_{ce}\,\bar\epsilon\gamma_d\partial_e\chi - 40\,\partial_b F_{cd}\partial_b\partial_c F_{ae}\,\bar\epsilon\gamma_d\partial_e\chi + \\
&+ 8\,\partial_b F_{cd}\partial_a\partial_c F_{be}\,\bar\epsilon\gamma_e\partial_d\chi + 40\,\partial_b F_{cd}\partial_b\partial_c F_{ae}\,\bar\epsilon\gamma_e\partial_d\chi + \\
&+ 8\,\partial_b F_{cd}\partial_a\partial_b F_{ce}\,\bar\epsilon\gamma_e\partial_d\chi - 8\,\partial_b F_{ac}\partial_b\partial_c F_{de}\,\bar\epsilon\gamma_d\partial_e\chi + \\
&+ 16\,\partial_b F_{ac}\partial_b\partial_d F_{ce}\,\bar\epsilon\gamma_d\partial_e\chi - 8\,\partial_b F_{ac}\partial_c\partial_d F_{be}\,\bar\epsilon\gamma_d\partial_e\chi + \\
&+ 4\,F_{bc}\partial_a\partial_b\partial_d F_{ef}\,\bar\epsilon\gamma_{cef}\partial_d\chi - 8\,\partial_b F_{cd}\partial_b\partial_c F_{ef}\,\bar\epsilon\gamma_{ade}\partial_f\chi + \\
&+ 8\,\partial_b F_{cd}\partial_c\partial_e F_{bf}\,\bar\epsilon\gamma_{ade}\partial_f\chi + 4\,\partial_b F_{cd}\partial_b\partial_c F_{ef}\,\bar\epsilon\gamma_{aef}\partial_d\chi + \\
&+ 6\,\partial_b F_{cd}\partial_a\partial_b F_{ef}\,\bar\epsilon\gamma_{cde}\partial_f\chi + 4\,\partial_b F_{cd}\partial_a\partial_b F_{ef}\,\bar\epsilon\gamma_{cef}\partial_d\chi + \\
&+ 2\,\partial_b F_{cd}\partial_a\partial_e F_{bf}\,\bar\epsilon\gamma_{cdf}\partial_e\chi + 4\,\partial_b F_{cd}\partial_b\partial_e F_{fg}\,\bar\epsilon\gamma_{acdfg}\partial_e\chi + \\
&+ 16\,F_{bc}\partial_b\partial_d F_{ae}\,\bar\epsilon\gamma_e\partial_c\partial_d\chi - 56\,F_{bc}\partial_a\partial_d F_{be}\,\bar\epsilon\gamma_e\partial_c\partial_d\chi + \\
&+ 24\,F_{ab}\partial_b\partial_c F_{de}\,\bar\epsilon\gamma_d\partial_c\partial_e\chi - 80\,F_{bc}\partial_a\partial_b F_{de}\,\bar\epsilon\gamma_d\partial_c\partial_e\chi - \\
&- 4\,F_{bc}\partial_d\partial_e F_{bc}\,\bar\epsilon\gamma_a\partial_d\partial_e\chi + 84\,F_{bc}\partial_b\partial_d F_{ae}\,\bar\epsilon\gamma_c\partial_d\partial_e\chi - \\
&- 116\,F_{bc}\partial_d\partial_e F_{ab}\,\bar\epsilon\gamma_c\partial_d\partial_e\chi + 24\,\partial_b F_{cd}\partial_b F_{ce}\,\bar\epsilon\gamma_a\partial_d\partial_e\chi - \\
&- 16\,\partial_b F_{cd}\partial_c F_{be}\,\bar\epsilon\gamma_a\partial_d\partial_e\chi + 8\,\partial_b F_{cd}\partial_b F_{ce}\,\bar\epsilon\gamma_e\partial_a\partial_d\chi - \\
&- 8\,\partial_b F_{cd}\partial_c F_{be}\,\bar\epsilon\gamma_e\partial_a\partial_d\chi + 64\,\partial_b F_{cd}\partial_a F_{ce}\,\bar\epsilon\gamma_e\partial_b\partial_d\chi + \\
&+ 8\,\partial_b F_{cd}\partial_c F_{ae}\,\bar\epsilon\gamma_e\partial_b\partial_d\chi - 16\,\partial_b F_{ac}\partial_b F_{de}\,\bar\epsilon\gamma_d\partial_c\partial_e\chi + \\
&+ 56\,\partial_b F_{ac}\partial_c F_{de}\,\bar\epsilon\gamma_d\partial_b\partial_e\chi - 16\,\partial_b F_{ac}\partial_d F_{be}\,\bar\epsilon\gamma_e\partial_c\partial_d\chi - \\
&- 8\,\partial_b F_{ac}\partial_d F_{ce}\,\bar\epsilon\gamma_e\partial_b\partial_d\chi - 4\,F_{ab}\partial_c\partial_d F_{ef}\,\bar\epsilon\gamma_{bef}\partial_c\partial_d\chi - \\
&- 2\,F_{bc}\partial_a\partial_d F_{ef}\,\bar\epsilon\gamma_{bce}\partial_d\partial_f\chi - 8\,F_{bc}\partial_b\partial_d F_{ef}\,\bar\epsilon\gamma_{ace}\partial_d\partial_f\chi - \\
&- 8\,F_{bc}\partial_d\partial_e F_{bf}\,\bar\epsilon\gamma_{acf}\partial_d\partial_e\chi - 4\,F_{bc}\partial_a\partial_d F_{ef}\,\bar\epsilon\gamma_{bef}\partial_c\partial_d\chi - \\
&- 4\,\partial_a F_{bc}\partial_d F_{ef}\,\bar\epsilon\gamma_{bce}\partial_d\partial_f\chi - 4\,\partial_a F_{bc}\partial_d F_{ef}\,\bar\epsilon\gamma_{bef}\partial_c\partial_d\chi + \\
&+ 4\,\partial_b F_{cd}\partial_b F_{ef}\,\bar\epsilon\gamma_{cde}\partial_a\partial_f\chi + 4\,\partial_b F_{cd}\partial_e F_{bf}\,\bar\epsilon\gamma_{acd}\partial_e\partial_f\chi + \\
&+ 8\,\partial_b F_{cd}\partial_c F_{ef}\,\bar\epsilon\gamma_{ade}\partial_b\partial_f\chi + 4\,F_{bc}\partial_d\partial_e F_{fg}\,\bar\epsilon\gamma_{abcfg}\partial_d\partial_e\chi + \\
&+ 2\,\partial_b F_{cd}\partial_e F_{fg}\,\bar\epsilon\gamma_{acdfg}\partial_b\partial_e\chi + 16\,F_{ab}\partial_c F_{de}\,\bar\epsilon\gamma_d\partial_b\partial_c\partial_e\chi + \\
&+ 32\,F_{bc}\partial_d F_{be}\,\bar\epsilon\gamma_a\partial_c\partial_d\partial_e\chi + 4\,F_{bc}\partial_d F_{ef}\,\bar\epsilon\gamma_{bce}\partial_a\partial_d\partial_f\chi\}\,. \qquad \text{(A.1)}
\end{aligned}
$$

The variations of the fermion are:

$$
\begin{aligned}
\delta\chi \,=\, & \tfrac{1}{2}\gamma_{ab}\epsilon F_{ab} + \frac{a_{(2,0)}\alpha'^2}{32}\Big\{ + F_{ab}F_{cd}F_{cd}\,\gamma_{ab}\epsilon - 4\,F_{ab}F_{cd}F_{ac}\,\gamma_{bd}\epsilon + \\
& \qquad + \frac{1}{6}\,F_{ab}F_{cd}F_{ef}\,\gamma_{abcdef}\epsilon\Big\} + \\
& + \frac{a_{(2,0)}^2\alpha'^2}{1024}\Big\{ - 12\,F_{ab}F_{ac}F_{bd}F_{cd}F_{ef}\,\gamma_{ef}\epsilon + F_{ab}F_{ab}F_{cd}F_{cd}F_{ef}\,\gamma_{ef}\epsilon - \\
& \qquad - 8\,F_{ab}F_{ab}F_{cd}F_{ce}F_{df}\,\gamma_{ef}\epsilon + 64\,F_{ab}F_{ac}F_{bd}F_{ce}F_{df}\,\gamma_{ef}\epsilon +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{3}\,F_{ab}F_{ab}F_{cd}F_{ef}F_{gh}\,\gamma_{cdefgh}\epsilon - 4\,F_{ab}F_{cd}F_{ae}F_{bf}F_{gh}\,\gamma_{cdefgh}\epsilon + \\
&+\frac{1}{60}\,F_{ab}F_{cd}F_{ef}F_{gh}F_{ij}\,\gamma_{abcdefghij}\epsilon\Big\} + \\
+a_{(4,4)}\alpha'^4\{&-8\,\gamma_{ab}\epsilon\,F_{ac}\partial_d F_{ef}\partial_b\partial_c\partial_d F_{ef} - 16\,\gamma_{ab}\epsilon\,F_{cd}\partial_e F_{cf}\partial_d\partial_e\partial_f F_{ab} - \\
&-8\,\epsilon\,\partial_a F_{bc}\partial_b F_{de}\partial_a\partial_c F_{de} + 2\,\gamma_{ab}\epsilon\,F_{cd}\partial_e\partial_f F_{ab}\partial_e\partial_f F_{cd} + \\
&+72\,\gamma_{ab}\epsilon\,F_{cd}\partial_c\partial_e F_{af}\partial_e\partial_f F_{bd} - 40\,\gamma_{ab}\epsilon\,F_{cd}\partial_c\partial_e F_{af}\partial_d\partial_f F_{be} + \\
&+16\,\gamma_{ab}\epsilon\,F_{cd}\partial_c\partial_e F_{af}\partial_d\partial_e F_{bf} - 56\,\gamma_{ab}\epsilon\,F_{cd}\partial_e\partial_f F_{ac}\partial_e\partial_f F_{bd} + \\
&+24\,\gamma_{ab}\epsilon\,F_{ac}\partial_d\partial_e F_{bf}\partial_d\partial_f F_{ce} - 16\,\gamma_{ab}\epsilon\,F_{ac}\partial_d\partial_e F_{bf}\partial_d\partial_e F_{cf} - \\
&-2\,\gamma_{ab}\epsilon\,F_{ab}\partial_c\partial_d F_{ef}\partial_c\partial_d F_{ef} - 4\,\gamma_{abcd}\epsilon\,F_{ae}\partial_f\partial_g F_{bc}\partial_f\partial_g F_{de} - \\
&-4\,\gamma_{abcd}\epsilon\,F_{ae}\partial_f\partial_g F_{bc}\partial_e\partial_f F_{dg} - \gamma_{abcdef}\epsilon\,F_{ab}\partial_g\partial_h F_{cd}\partial_g\partial_h F_{ef} - \\
&-12\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_c F_{df}\partial_e\partial_f F_{ab} + 8\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_d F_{cf}\partial_e\partial_f F_{ab} + \\
&+64\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_d F_{af}\partial_c\partial_e F_{bf} - 48\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_f F_{ad}\partial_c\partial_e F_{bf} - \\
&-56\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_f F_{ad}\partial_c\partial_f F_{be} + 64\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_f F_{ad}\partial_e\partial_f F_{bc} - \\
&-16\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_d F_{af}\partial_e\partial_f F_{bc} - 8\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_d F_{af}\partial_c\partial_f F_{be} - \\
&-4\,\gamma_{ab}\epsilon\,\partial_c F_{ad}\partial_b F_{ef}\partial_c\partial_d F_{ef} - 16\,\gamma_{ab}\epsilon\,\partial_c F_{ad}\partial_e F_{bf}\partial_c\partial_f F_{de} - \\
&-6\,\gamma_{ab}\epsilon\,\partial_c F_{de}\partial_f F_{ab}\partial_c\partial_f F_{de} + 4\,\gamma_{abcd}\epsilon\,\partial_e F_{af}\partial_f F_{bg}\partial_e\partial_g F_{cd} - \\
&-4\,\gamma_{abcd}\epsilon\,\partial_e F_{af}\partial_g F_{bc}\partial_f\partial_g F_{de} + 16\,\gamma_{ab}\epsilon\,\partial_c F_{ad}\partial_e F_{bf}\partial_c\partial_e F_{df} - \\
&-\gamma_{abcdef}\epsilon\,\partial_g F_{ab}\partial_h F_{cd}\partial_g\partial_h F_{ef}\}\,. \qquad \text{(A.2)}
\end{aligned}
$$

A.2 Nonlinear supersymmetry

The nonlinear supersymmetry transformations of the vector field are:

$$
\begin{aligned}
\delta A_a = {}& a_{(2,0)}\alpha'^2\Big\{+\frac{1}{4}F_{ab}\,\bar\eta\gamma_b\chi - \frac{1}{8}F_{bc}\,\bar\eta\gamma_{abc}\chi\Big\} + \\
&+a^2_{(2,0)}\alpha'^4\Big\{-\frac{1}{768}F_{bc}F_{de}F_{fg}\,\bar\eta\gamma_{abcdefg}\chi + \frac{1}{128}F_{bc}F_{ad}F_{ef}\,\bar\eta\gamma_{bcdef}\chi + \\
&\qquad+\frac{3}{128}F_{bc}F_{bc}F_{de}\,\bar\eta\gamma_{ade}\chi + \frac{1}{16}F_{ab}F_{bc}F_{de}\,\bar\eta\gamma_{cde}\chi + \\
&\qquad+\frac{1}{32}F_{bc}F_{bd}F_{ce}\,\bar\eta\gamma_{ade}\chi - \frac{3}{64}F_{bc}F_{bc}F_{ad}\,\bar\eta\gamma_d\chi - \\
&\qquad-\frac{1}{16}F_{ab}F_{bc}F_{cd}\,\bar\eta\gamma_d\chi\Big\} + \qquad \text{(A.3)} \\
&+a_{(4,4)}\alpha'^4\{-20\,\partial_b\partial_c F_{ad}\,\bar\eta\gamma_d\partial_b\partial_c\chi + 12\,\partial_b\partial_c F_{ad}\,\bar\eta\gamma_b\partial_c\partial_d\chi - 4\,\partial_b\partial_c F_{de}\,\bar\eta\gamma_{ade}\partial_b\partial_c\chi\}\,.
\end{aligned}
$$

The nonlinear supersymmetry transformations of the fermion are:

$$
\begin{aligned}
\delta\chi = {}& \eta + a_{(2,0)}\alpha'^2\Big\{+\frac{1}{16}F_{ab}F_{ab}\,\eta + \frac{1}{32}F_{ab}F_{cd}\,\gamma_{abcd}\eta\Big\} + \\
&+a^2_{(2,0)}\alpha'^4\Big\{+\frac{1}{512}F_{ab}F_{ab}F_{cd}F_{cd}\,\eta - \frac{3}{128}F_{ab}F_{ac}F_{bd}F_{cd}\,\eta + \\
&\qquad+\frac{1}{512}F_{ab}F_{ab}F_{cd}F_{ef}\,\gamma_{cdef}\eta - \frac{1}{64}F_{ab}F_{ac}F_{bd}F_{ef}\,\gamma_{cdef}\eta +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{6144}F_{ab}F_{cd}F_{ef}F_{gh}\,\gamma_{abcdefgh}\eta\Big\}+\\
+a_{(4,4)}\alpha'^4\{&-4\,\partial_a\partial_bF_{cd}\partial_a\partial_bF_{cd}\,\eta+8\,\partial_a\partial_bF_{cd}\partial_a\partial_bF_{ce}\,\gamma_{de}\eta+\\
&-16\,\partial_a\partial_bF_{cd}\partial_a\partial_cF_{be}\,\gamma_{de}\eta-2\,\partial_a\partial_bF_{cd}\partial_a\partial_bF_{ef}\,\gamma_{cdef}\eta\}\,. \qquad \text{(A.4)}
\end{aligned}
$$

References

[1] A.A. Tseytlin, *Born-Infeld action, supersymmetry and string theory*, in *Yuri Golfand memorial volume*, M. Shifman ed., World Scientific 2000 [hep-th/9908105].

[2] E.S. Fradkin and A.A. Tseytlin, *Nonlinear electrodynamics from quantized strings*, *Phys. Lett.* **B 163** (1985) 123.

[3] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super three-brane in ten-dimensional type-IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148].

[4] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super-p-branes in ten-dimensional type-IIA and -IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159].

[5] E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[6] M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge-invariant and gauge-fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99 [hep-th/9612080].

[7] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *The structure of maximally supersymmetric Yang-Mills theory: constraining higher-order corrections*, *J. High Energy Phys.* **06** (2001) 034 [hep-th/0102009]; $D = 10$ *super-Yang-Mills at* α'^2, *J. High Energy Phys.* **07** (2001) 042 [hep-th/0104236].

[8] P. Koerber and A. Sevrin, *The non-abelian Born-Infeld action through order* α'^3, *J. High Energy Phys.* **10** (2001) 003 [hep-th/0108169].

[9] A. Refolli, A. Santambrogio, N. Terzi and D. Zanon, F^5 *contributions to the nonabelian Born-Infeld action from a supersymmetric Yang-Mills five-point function*, *Nucl. Phys.* **B 613** (2001) 64 [hep-th/0105277].

[10] D.T. Grasso, *Higher order contributions to the effective action of* $N = 4$ *super Yang-Mills*, *J. High Energy Phys.* **11** (2002) 012 [hep-th/0210146].

[11] A. Collinucci, M. De Roo and M.G.C. Eenink, *Supersymmetric Yang-Mills theory at order* α'^3, *J. High Energy Phys.* **06** (2002) 024 [hep-th/0205150].

[12] R. Medina, F.T. Brandt and F.R. Machado, *The open superstring 5-point amplitude revisited*, *J. High Energy Phys.* **07** (2002) 071 [hep-th/0208121].

[13] P. Koerber and A. Sevrin, *The non-abelian D-brane effective action through order* α'^4, *J. High Energy Phys.* **10** (2002) 046 [hep-th/0208044].

[14] E.A. Bergshoeff, M. de Roo and A. Sevrin, *Non-abelian Born-Infeld and kappa-symmetry*, *J. Math. Phys.* **42** (2001) 2872 [hep-th/0011018].

[15] E.A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, *Supersymmetric non-abelian Born-Infeld revisited*, *J. High Energy Phys.* **07** (2001) 029 [hep-th/0105274].

[16] D.P. Sorokin, *Coincident (super)-dp-branes of codimension one*, *J. High Energy Phys.* **08** (2001) 022 [hep-th/0106212].

[17] J.M. Drummond, P.S. Howe and U. Lindström, *Kappa-symmetric non-abelian Born-Infeld actions in three dimensions*, *Class. and Quant. Grav.* **19** (2002) 6477 [hep-th/0206148].

[18] S.S. Pal and S. Panda, *Coincident Dp-branes in codimension two*, hep-th/0211115.

[19] D.J. Gross and E. Witten, *Superstring modifications of Einstein's equations*, *Nucl. Phys.* **B 277** (1986) 1.

[20] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *Spinorial cohomology of abelian $d = 10$ super-Yang-Mills at $\mathcal{O}(\alpha'^3)$*, *J. High Energy Phys.* **11** (2002) 023 [hep-th/0205165].

[21] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of Möbius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205.

[22] N. Wyllard, *Derivative corrections to D-brane actions with constant background fields*, *Nucl. Phys.* **B 598** (2001) 247 [hep-th/0008125].

[23] N. Wyllard, *Derivative corrections to the D-brane Born-Infeld action: non-geodesic embeddings and the Seiberg-Witten map*, *J. High Energy Phys.* **08** (2001) 027 [hep-th/0107185].

[24] S.R. Das, S. Mukhi and N.V. Suryanarayana, *Derivative corrections from noncommutativity*, *J. High Energy Phys.* **08** (2001) 039 [hep-th/0106024].

[25] J.H. Schwarz, *Superstring theory*, *Phys. Rept.* **89** (1982) 223.

[26] A. Bilal, *Higher-derivative corrections to the non-abelian Born-Infeld action*, *Nucl. Phys.* **B 618** (2001) 21 [hep-th/0106062].

[27] A. Van Proeyen, *Tools for supersymmetry*, hep-th/9910030.

[28] E. Bergshoeff, M. Rakowski and E. Sezgin, *Higher derivative super Yang-Mills theories*, *Phys. Lett.* **B 185** (1987) 371.